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DISCRETE MODEL FOR DYNAMIC THROUGH-THE-SOIL COUPLING OF 3-D FOUNDATIONS AND STRUCTURES

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SUMMARY

An efficient discrete model for predicting the dynamic through-the-soil interaction between adjacent rigid, surface foundations supported by a homogeneous, isotropic and linear elastic half-space is presented. The model utilizes frequency-independent springs and dashpots, and the foundation mass, for the consideration of soil–foundation interaction. The through-the-soil coupling of the foundations is attained by frequency-independent stiffness and damping functions, developed in this work, that interconnect the degrees of freedom of the entire system of foundations. The dynamic analysis of the resulting coupled system is performed in the time domain and includes the time lagging effects of coupled dynamic input due to wave propagation using an appropriate modification of the Wilson-θ method. The basic foundation interaction model is also extended to the evaluation of coupled building-foundation systems. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: foundation–soil–foundation interaction; discrete model; soil–structure interaction

INTRODUCTION

It is broadly accepted nowadays, after several theoretical, experimental and field studies, that the dynamic response of structural systems is significantly affected by the interaction with adjacent structures, in addition, of course, to several other well-known factors. Even the dynamic response of the independent or structurally interconnected adjacent foundations of a single structure is affected by through-the-soil dynamic coupling among these foundations and, thus, the dynamic behaviour of the entire structure is influenced. This phenomenon, rarely studied only during the last 30 years, is generally considered to fall within the Soil–Structure Interaction (SSI) field although it clearly presents interactive characteristics beyond those of the classic, and well understood, Foundation–Soil Interaction (FSI) problem.

The computation of dynamic interaction between adjacent foundations and structures requires the solution of the FSI problem and the problem of foundation coupling through the supporting medium. Solutions to either one of these problems can be obtained using rigorous analytical formulations, which usually pertain to simple geometries and loading functions, or numerical procedures, usually FEM or BEM, for more complicated configurations. In addition, a limited number of approximate discrete models with relatively few Degrees Of Freedom (DOFs) have also been developed. These models provide practical and efficient alternatives to computationally expensive numerical solutions or mathematically complicated analytical formulations since their usage requires the application of basic concepts of structural dynamics and access to a modest computer. While their order of accuracy is considered inferior to that of the more rigorous analytic or numerical solutions it usually falls within the range of accuracy expected for

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a soil–foundation system which by its nature cannot be accurately defined. The focus of this work is on the development of a simple discrete model for dynamic 3-D Foundation–Soil–Foundation Interaction (FSFI) analysis based on results obtained by more precise numerical investigations. Therefore, in the following, specific reference is made to those published works which advanced the development of such discrete models. The interested reader can find updated literature reviews on analytic and numerical solutions of various FSFI problems in the recent articles of Qian and Beskos,[1,2] and Mohammadi and Karabalis.[3]

One of the earliest efforts to develop practical discrete models for dynamic FSI was undertaken by Lysmer and Richart.[4] Their investigation resulted in a simple Single-Degree-Of-Freedom (SDOF) model for computing the forced vertical response of foundations attached to an elastic half-space. This concept of a lumped-parameter representation of foundations on a half-space is described in great detail and further developed by Whitman,[5] Richart *et al.*,[6] Newmark and Rosenblueth,[7] Gazetas[8] and Wolf,[9,10] among several others. Meek and Veletsos[11] extended these basic models by introducing an additional DOF to the dynamic system. Generalizations of this method were developed by De Barros and Luco[12] for surface and embedded foundations and Wolf and Somaini[13] for embedded foundations. Wolf and Paronesso[14,15] used Multiple-Degree-Of-Freedom (MDOF) lumped-parameter models for surface and embedded foundations on/in a soil layer over bedrock. Wolf[16,17] has also presented a set of 'consistent' lumped-parameter models for soil media.

The through-the-soil coupling of foundations was first introduced by Whitman[5] as an important problem requiring study. Among the first publications in this area one should mention those of Warburton *et al.*,[18,19] who performed studies of two masses with identical circular bases attached to an elastic half-space, and MacCalden and Matthiesen,[20] who developed a matrix formulation for the solution of the induced dynamic displacement of a foundation near a harmonically loaded foundation attached to an elastic half-space. However, comparison studies, presented in the latter publication, between theoretical and experimental results showed significant discrepancies. A special mention should also be made to the mathematically rigorous solutions presented by Triantafyllidis[21] and Triantafyllidis and Prange[22] which, however, are unavoidably restricted to specific geometries. Among the latest numerical solutions one can mention the time-domain BEM approach of Guan and Novak[23] based on the half-space solutions provided by the same authors,[24] and the frequency-domain BEM formulation of Qian and Beskos.[1,2] In both cases half-space solutions have been used requiring the discrertization of only the contact surface between the soil medium and the foundation. Huang[25] and Karabalis and Huang[26] have also reported on a time-domain solution of the 3-D FSFI problem using the BEM in conjunction with the Stokes fundamental solutions. Soil layering along with FSFI was studied by Karabalis and Mohammadi[27,28] using a frequency-domain BEM and full-space fundamental solutions. Thus, their solution required the discretization of the interlayer contact surface in addition to that of the free surface. The dynamic structure–soil–structure interaction problem has been studied analytically by Luco and Contesse,[29] Wong and Trifunac[30] and Murakami and Luco.[31] In these articles the loading functions were obliquely incident SH waves and the structures under consideration consisted of two or more shear walls founded on semicylindrical rigid foundations. Their numerical studies showed that groups of closely spaced buildings can result in interaction effects near the fundamental frequencies of the buildings and at very low frequencies. Structural coupling, in addition to through-the-soil coupling, between adjacent foundations has been considered by Mohammadi and Karabalis[3] for the case of a railway line using a frequency-domain BEM/FEM formulation. Their system of up to nine ties interconnected by rails showed significant sensitivity to both structural and through-the-soil coupling the influence of which was studied independently. An approximate analytical–numerical approach was proposed by Lee and Wesley[32] for the solution of a 3-D through-the-soil interaction problem involving three rigid circular foundations and spring-mass models for the superstructures attached to the foundations.

In the work described in this paper, coupling functions corresponding to discrete springs and dashpots have been developed for the computation of the dynamic interaction of massive surface foundations supported by a homogeneous, isotropic, linear elastic half-space. The time-lagging effects associated with wave propagation are taken into consideration via a modification of the Wilson-θ method[33] developed for

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the needs of this work. For the solution of the associated FSI problem, discrete models along the lines of those proposed by Richart *et al.*,[6] Gazetas[8] and Wolf[9,10] have been adopted, while the development of the coupling functions is based on the numerical results presented by Huang.[25] The dynamic response of a number of foundations due to external forces applied to one foundation is computed directly in the time domain utilizing the coupling functions. These functions are also used, in the 'numerical results' section, to compute through-the-soil coupling effects of closely spaced buildings subjected to ground acceleration. The employment of such a direct time-domain analysis has the additional advantage of enabling the consideration of non-linear soil and structures by simple modifications of the basic models described herein.

The original work of Mulliken[34] is the basis of this article while later developments and results produced by the authors have also been incorporated. Parts of this work have also been presented in Mulliken and Karabalis.[35]

FOUNDATION–SOIL–FOUNDATION SYSTEM

The geometry and nomenclature for the foundation–soil–foundation system studied in this work are as shown in Figure 1. This system can be viewed as an assemblage of individual unconstrained foundation–soil systems coupled by the, common to all of them, soil medium. The problem is solved by first defining the individual FSI systems with discrete masses, springs and dampers where each mode of vibration is considered as an independent DOF, as shown in Figure 2. Subsequently, foundation coupling due to wave propagation is introduced by an approximate method developed by Mulliken.[34] This method introduces frequency-independent coupling functions developed for each foundation DOF which result in springs and dashpots that link the various DOFs of the foundations, as shown in Figure 3. The incorporation of these springs and

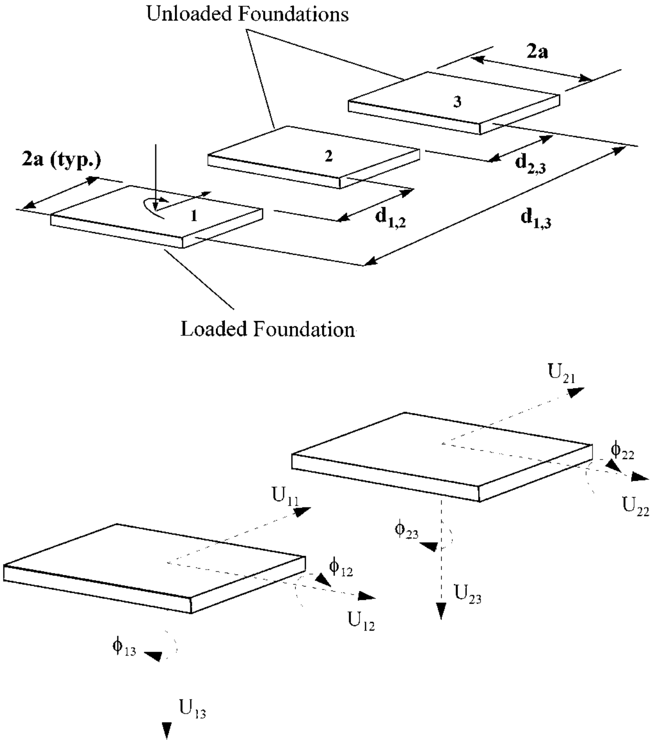

Figure 1. Foundation system geometry and nomenclature

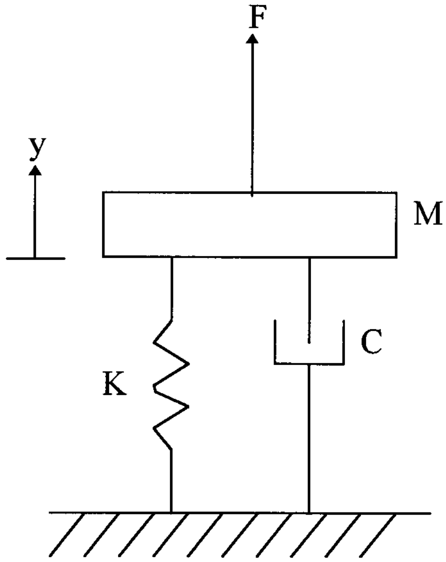

Figure 2. Basic one-dimensional soil–foundation discrete model

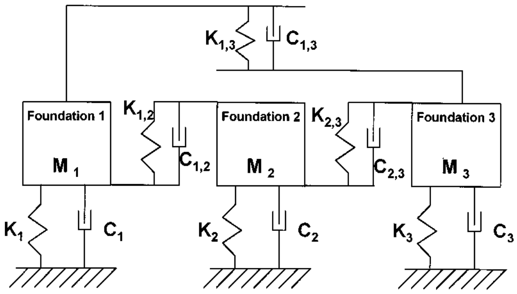

Figure 3. Foundation coupling model

dashpots into the solution of a coupled dynamic system is achieved by a proposed modification of the Wilson-θ method so that time-lagging effects due to wave propagation are also taken into consideration.

The following three assumptions underline the development of the coupling model described in this work: (a) coupling between foundations is determined for each mode of vibration independent of the other DOFs, i.e. vertical motions at one foundation produce only vertical responses at the adjacent foundations, rocking input produces only rocking responses, etc., (b) the static stiffness of each individual foundation–soil system is not significantly effected by the close proximity of the adjacent foundations (an exception is made later for the rocking motion), and (c) the coupling forces at one foundation occur at some time after the displacement of the adjacent foundation, and the time lag is directly related to the wave speeds in the supporting medium. Given these assumptions, the static stiffness and damping of individual foundation–soil systems are stored in one set of matrices, while the coupling stiffness and damping are maintained as off-diagonal terms in separate matrices. This is essential to the FSFI problem investigated in this work since, as it turns out, the presence of the nearby foundations, modelled by the coupling terms, does not influence (except for the rocking motion in the very near field) the individual stiffness and damping of single foundations. In addition, if constant stiffness and damping matrices with off-diagonal terms were used, it would introduce direct coupling between DOFs,

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and time-lagging effects between motions of adjacent foundations would not be possible. However, this work evidences the fact that ignoring this time lag significantly alters the characteristics (frequency content and amplitude) of motion in the time domain.

In the following, first the frequency-independent FSI models utilized in this work will be discussed. Next, the time-domain solution of the forced dynamic response of single-degree-of-freedom FSI systems via the Wilson-θ method is presented for completeness. The proposed modifications to the Wilson-θ method required in order to compute coupling effects between multiple, adjacent foundations is then introduced. Given these concepts the development of frequency-independent coupling functions is then described. As it will be exhibited, these coupling functions enable the computation of dynamic through-the-soil coupling between any number of square foundations for various time-dependent loading functions. Although the basic concepts of the Wilson-θ method are utilized in this work, any other direct integration method of preference can be used, in concert with the described modifications, to evaluate foundation coupling.

Discrete FSI models

The relationship between an applied loading function on a rigid, massive foundation attached to an elastic medium, and the corresponding displacement function of the foundation, can be determined for each foundation DOF using a simple SDOF lumped parameter model, as shown in Figure 2. The mass, M, represents the mass (or rotational inertia) of the foundation, m, plus a virtual soil mass (or rotational inertia), m_v, attached to the foundation. This total mass is supported by a rigid support through a spring having a stiffness, K, and a dashpot having a coefficient, C. These three parameters should be defined for each DOF.

Newmark and Rosenblueth[7] describe the concept of virtual soil mass as a simple approach to the consideration of the soil mass contribution to FSI problems. This method enables the use of frequency-independent discrete models for the solution of a strongly frequency-dependent problem. After some numerical experimentation with various combinations of available coefficients for M, K, and C the virtual soil mass coefficients adopted in this work are those proposed by Richart *et al.*[6] and Gazetas,[8] while for the stiffness those of the Wolf[10] model are used. The damping coefficients while they are in the form proposed by Wolf[10] have been adapted so that in combination with the corresponding coefficients for mass and stiffness produce the best fit to the various available data for the SDOF soil–foundation system. Table I below reproduces the equations used in the computation of these coefficients with a being the half-width of a square foundation and $V_s = \sqrt{G/\rho}$ denoting the shear wave velocity, where G and ρ are the shear modulus and mass density, respectively, of the soil medium.

Table I. Mass, stiffness and damping coefficients for one-dimensional discrete FSI model

	Mass (inertia) ratio, β	Equivalent radius, r_0	Virtual soil mass (inertia), m_v	Static stiffness K	Damping C
Vertical	$\frac{(1-v)}{4}\frac{m}{\rho r_0^3}$	$\frac{2a}{\sqrt{\pi}}$	$\frac{0{\cdot}27m}{\beta}$	$\frac{4{\cdot}7Ga}{1-v}$	$\frac{0{\cdot}8a}{V_s}K$
Horizontal	$\frac{(7-8v)}{32(1-v)}\frac{m}{\rho r_0^3}$	$\frac{2a}{\sqrt{\pi}}$	$\frac{0{\cdot}095m}{\beta}$	$\frac{9{\cdot}2Ga}{2-v}$	$\frac{0{\cdot}163a}{V_s}K$
Rocking	$\frac{3(1-v)}{8}\frac{m}{\rho r_0^5}$	$\frac{2a}{\sqrt[4]{3\pi}}$	$\frac{0{\cdot}24m}{\beta}$	$\frac{4{\cdot}0Ga^3}{1-v}$	$\frac{0{\cdot}6a}{V_s}K$
Torsion	$\frac{m}{\rho r_0^5}$	$\frac{2a}{\sqrt[4]{3\pi}}$	$\frac{0{\cdot}045m}{\beta}$	$8{\cdot}31Ga^3$	$\frac{0{\cdot}127a}{V_s}K$

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In view of the above definitions, the equation of motion for each DOF of a single foundation is

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F(t) \tag{1}$$

where $\ddot{y}(t)$, $\dot{y}(t)$ and $y(t)$ are the acceleration, velocity and displacement, respectively, for the DOF under consideration, and $F(t)$ is the externally applied forcing function.

Time-domain formulation—SDOF system

The step-by-step linear acceleration method with the Wilson-θ extended step is used for the integration of the equation of motion (1). The basic assumption of the Wilson-θ method, as shown in Figure 4, is that the acceleration in the equation of motion can be represented by a linear function during the time step $\tau = \theta\Delta t$. By extending the time step, Δt, by the factor θ, the numerical stability of the solution is assured regardless of the time step chosen as long as $\theta \geqslant 1{\cdot}38$. In the following, the Wilson-θ method is outlined, for completeness purposes, on the basis of the description provided by Paz.[33]

From Figure 4, it is seen that the area under the acceleration line over the extended step ($t = t_i + \tau$), i.e. the incremental velocity, is evaluated by

$$\hat{\Delta}\dot{y}_i = \ddot{y}_i\tau + \tfrac{1}{2}\hat{\Delta}\ddot{y}_i\tau \tag{2}$$

and, thus, the incremental displacement is

$$\hat{\Delta}y_i = \dot{y}_i\tau + \tfrac{1}{2}\ddot{y}_i\tau^2 + \tfrac{1}{6}\hat{\Delta}\ddot{y}_i\tau^2 \tag{3}$$

Solving equation (3) for the incremental acceleration $\hat{\Delta}\ddot{y}_i$ and substituting in equation (2) yields

$$\hat{\Delta}\ddot{y}_i = \alpha_4\hat{\Delta}y_i - \alpha_2\dot{y}_i - 3\ddot{y}_i \tag{4}$$

$$\hat{\Delta}\dot{y}_i = \alpha_1\hat{\Delta}y_i - 3\dot{y}_i - \alpha_3\ddot{y}_i \tag{5}$$

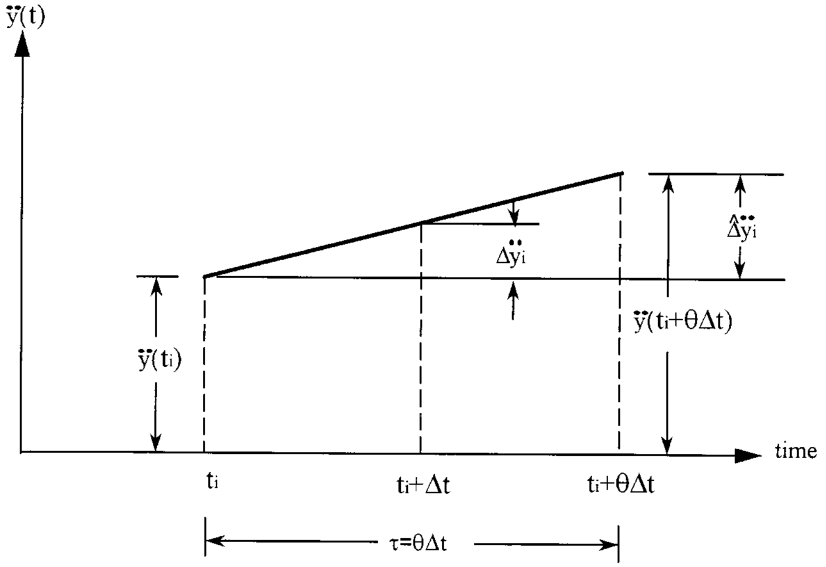

Figure 4. Linear acceleration assumption in the Wilson extended time interval

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where

$$\alpha_1 = \frac{3}{\tau}, \quad \alpha_2 = \frac{6}{\tau}, \quad \alpha_3 = \frac{\tau}{2}, \quad \alpha_4 = \frac{6}{\tau^2} \tag{6}$$

Taking the difference between the equilibrium conditions defined at t_i and $t_i + \tau$, the dynamic equation of motion (1) can be expressed in incremental form as

$$M\hat{\Delta}\ddot{y}_i + C\hat{\Delta}\dot{y}_i + K\hat{\Delta}y_i = \hat{\Delta}F_i \tag{7}$$

where the incremental load $\hat{\Delta}F_i$ is determined by linear interpolation as

$$\hat{\Delta}F_i = F_{i+1} + (F_{i+2} - F_{i+1})\cdot(\theta - 1) - F_i \tag{8}$$

Direct substitution of equations (4) and (5) into this incremental equation of motion (7) results in an equation for the incremental displacement which can be written conveniently as

$$\bar{K}\hat{\Delta}y_i = \hat{\Delta}\bar{F}_i \tag{9}$$

where the effective stiffness matrix is

$$\bar{K} = K + \alpha_4 M + \alpha_1 C \tag{10}$$

and the effective incremental load is

$$\hat{\Delta}\bar{F}_i = \hat{\Delta}F_i + (\alpha_2 M + 3C)\dot{y}_i + (3M + \alpha_3 C)\ddot{y}_i \tag{11}$$

The incremental displacement for the extended time interval is obtained from equation (9) which upon substitution into equation (4) yields the incremental acceleration for the extended step. Subsequently, the incremental acceleration $\Delta\ddot{y}_i$ for the normal time increment Δt is determined by linear interpolation as

$$\Delta\ddot{y}_i = \frac{\hat{\Delta}\ddot{y}_i}{\theta} \tag{12}$$

After computing the incremental acceleration, the displacement and velocity at the end of the time interval $t_i + \Delta t$ can be computed by direct substitution into equations (2) and (3) using the normal time increment, while the corrected acceleration is computed directly from the equation of motion (1) at $t = t_{i+1}$, i.e.,

$$\dot{y}_{i+1} = \dot{y}_i + \ddot{y}_i \Delta t + \frac{1}{2}\Delta\ddot{y}_i \Delta t \tag{13}$$

$$y_{i+1} = y_i + \dot{y}_i \Delta t + \frac{1}{2}\ddot{y}_i \Delta t^2 + \frac{1}{6}\Delta\ddot{y}_i \Delta t^2 \tag{14}$$

$$\ddot{y}_{i+1} = M^{-1}(F_{i+1} - C\dot{y}_{i+1} - Ky_{i+1}) \tag{15}$$

The Wilson-θ procedure, outlined above, can, of course, be repeatedly used to compute the values of acceleration, velocity and displacement at subsequent time steps until the desired final time of the analysis is reached.

Through-the-soil coupling of adjacent foundations

The through-the-soil coupling of adjacent foundations, as shown in Figure 1, is achieved via frequency-independent stiffness and damping coefficients, as shown in Figure 3. As previously explained, the basis of the required modification to Wilson-θ method is that the static stiffness of the individual foundations is essentially unchanged by the presence of other foundations, and that a finite time period exists between the response of one foundation and the influence of that response on all other foundations adjacent to it.

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The formulation described above for a single FSI system can be expanded to include multiple FSI systems by resorting to matrix notation. Thus, if no consideration is given to through-the-soil coupling effects the stiffness, mass and damping matrices of a group of independent FSI systems can be written in the form of diagonal matrices, with the subscripts indicating each foundation, as

$$[K]=\begin{bmatrix} K_1 & 0 & 0 & \cdots \\ 0 & K_2 & 0 & \cdots \\ 0 & 0 & K_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},\quad [M]=\begin{bmatrix} M_1 & 0 & 0 & \cdots \\ 0 & M_2 & 0 & \cdots \\ 0 & 0 & M_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},\quad [C]=\begin{bmatrix} C_1 & 0 & 0 & \cdots \\ 0 & C_2 & 0 & \cdots \\ 0 & 0 & C_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{16}$$

and, similarly, the displacement, velocity, acceleration, forcing function vectors as

$$y=\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{Bmatrix},\quad \dot{y}=\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \vdots \end{Bmatrix},\quad \ddot{y}=\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \\ \vdots \end{Bmatrix},\quad F=\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \end{Bmatrix}, \tag{17}$$

The inclusion of through-the-soil coupling is achieved by the introduction of stiffness and damping coefficients that inter-connect the individual FSI systems, as shown in Figure 3. These coefficients are stored as off-diagonal terms in a separate set of matrices, i.e. independent to those shown in equation (16), which are of the form

$$[K_c]=\begin{bmatrix} 0 & -K_{c(1,2)} & -K_{c(1,3)} & \cdots \\ -K_{c(2,1)} & 0 & -K_{c(2,3)} & \cdots \\ -K_{c(3,1)} & -K_{c(3,2)} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},\quad [C_c]=\begin{bmatrix} 0 & -C_{c(1,2)} & -C_{c(1,3)} & \cdots \\ -C_{c(2,1)} & 0 & -C_{c(2,3)} & \cdots \\ -C_{c(3,1)} & -C_{c(3,2)} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{18}$$

The subscripts here indicate the foundations that the coupling terms connect. The assembled matrices should be symmetrical since the reciprocity condition $K_{c(i,j)}=K_{c(j,i)}$ and $C_{c(i,j)}=C_{c(j,i)}$ holds for the coupling terms. The computation of these coupling terms is the subject of the next section.

The separation of the stiffness and damping matrices into non-coupling, equation (16), and coupling, equation (18), coefficients becomes necessary for two reasons. The first reason is that the effective stiffness matrix in equation (10) should not be altered by the presence of adjacent foundations. The second reason is that the coupling forces occur at some time after each foundation responds, e.g. if foundation i undergoes displacement, velocity and acceleration responses at time t, foundation j will be affected by those responses at some time $t+\Delta T$, as depicted in Figure 5, ΔT being the time lag for a coupled response. Thus, at any time during the analysis, the response of a foundation is the result of applied (external) and internal (SSI) forces at the current time, and forces due to coupling between foundations that are caused by the response of adjacent foundations at past time. By storing the coupling terms into separate matrices, it becomes possible to include in the forces applied in the current time step at each individual foundation those coupling forces that are due to previous responses at each adjacent foundation.

The actual time lag ΔT for the various coupled motions considered in this work are listed in Table II. These formulae have been derived from observations on numerical results presented by Huang[25] for a study on two adjacent rigid, surface square foundations using a time-domain BEM. As it was expected ΔT depends on the distance $d_{i,j}$ between foundations, the foundation half-width a, and the corresponding wave velocity.

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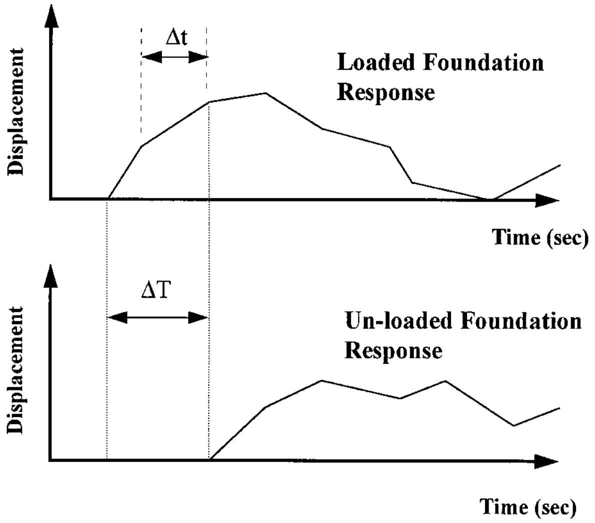

Figure 5. Time lag in coupled response between loaded and unloaded foundation

Table II. Time lag for coupled dynamic response between two adjacent, rigid, surface, square foundations

	Vertical motion	Horizontal motion	Rocking motion	Torsional motion
ΔT (sec)	$\frac{3}{4}\frac{(d+a/2)}{V_S}$	$\frac{(d+a/2)}{V_p}$	$\frac{(d+a/2)}{V_S}$	$\frac{(d+a/2)}{V_S}$

V_P in Table II is the compression wave, or P-wave, velocity defined as

$$V_P = V_S\sqrt{\frac{2-2v}{1-2v}} \tag{19}$$

with v being the Poisson ratio.

The incorporation of coupling between foundations in the Wilson-θ method involves modifications to the effective incremental load in equation (11) and the computation of acceleration at the end of the normal time increment in equation (15). The matrices $[\tilde{y}]_i$, $[\tilde{\dot{y}}]_i$ and $[\tilde{\ddot{y}}]_i$ introduced here are a collection of displacement, velocity and acceleration vectors, respectively, of each foundation at times that lag the current time t_i by ΔT and are of the form

$$[\tilde{y}]_i = \left[\begin{Bmatrix} 0 \\ y_2(t_i-\Delta T_{1,2}) \\ y_3(t_i-\Delta T_{1,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} y_1(t_i-\Delta T_{2,1}) \\ 0 \\ y_3(t_i-\Delta T_{2,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} y_1(t_i-\Delta T_{3,1}) \\ y_2(t_i-\Delta T_{3,2}) \\ 0 \\ \vdots \end{Bmatrix} \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \ddots \end{Bmatrix}\right] \tag{20}$$

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$$[\tilde{\dot{y}}]_i = \left[\begin{Bmatrix} 0 \\ \dot{y}_2(t_i - \Delta T_{1,2}) \\ \dot{y}_3(t_i - \Delta T_{1,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} \dot{y}_1(t_i - \Delta T_{2,1}) \\ 0 \\ \dot{y}_3(t_i - \Delta T_{2,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} \dot{y}_1(t_i - \Delta T_{3,1}) \\ \dot{y}_2(t_i - \Delta T_{3,2}) \\ 0 \\ \vdots \end{Bmatrix} \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \ddots \end{Bmatrix} \right] \tag{21}$$

$$[\tilde{\ddot{y}}] = \left[\begin{Bmatrix} 0 \\ \ddot{y}_2(t_i - \Delta T_{1,2}) \\ \ddot{y}_3(t_i - \Delta T_{1,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} \ddot{y}_1(t_i - \Delta T_{2,1}) \\ 0 \\ \ddot{y}_3(t_i - \Delta T_{2,3}) \\ \vdots \end{Bmatrix} \begin{Bmatrix} \ddot{y}_1(t_i - \Delta T_{3,1}) \\ \ddot{y}_2(t_i - \Delta T_{3,2}) \\ 0 \\ \vdots \end{Bmatrix} \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \ddots \end{Bmatrix} \right] \tag{22}$$

As an example, the first vector of the displacement matrix in equation (20) consists of the displacements of all foundations, with the exception of foundation 1, which are lagged based on the distance between each foundation and foundation 1, as noted in the subscripts referenced in the time lag, $\Delta T_{1,j}$. These displacements are required in order to determine the effect that prior displacements at all foundations have on foundation 1 in the current time increment. The second vector provides the lagged displacement input for all foundations that effect the response of foundation 2, and so on. The number of vectors is determined by the number of foundations. The same notation is used in the assembly of the time-lagged velocity, equation (21), and acceleration, equation (22), matrices.

The computation of coupling forces which is accomplished by multiplying the stiffness and damping coupling matrices in equation (18) by the time-lagged displacement, velocity and acceleration matrices in equations (20)–(22), requires special treatment. To produce *vectors* of force by the multiplication of two matrices the following formula, shown symbolically, is used

$$\{\tilde{F}\}_i = \sum_{k=1}^{n} a_{i,k} \cdot b_{k,i} \tag{23}$$

where $a_{i,k}$ represents the stiffness or damping coupling matrix, $b_{k,i}$ represents the time-lagged displacement, velocity or acceleration matrix, and n is the number of foundations. Thus, an application of equation (23) yields the forcing vector at a foundation based on the prior response of all foundations adjacent to it.

After the assemblage of the coupling matrices of equations (18), and the time-lagged matrices of equations (20)–(22), the computation of the effective incremental load, including through-the-soil coupling forces, can be achieved by modifying equation (11) as

$$\{\Delta\hat{F}\}_i = \{\Delta\hat{F}\}_i + (\alpha_2[M] + 3[C])\{\dot{y}\}_i + 3[C_c][\tilde{\dot{y}}]_i + (3[M] + \alpha_3[C])\{\ddot{y}\}_i + \alpha_3[C_c][\tilde{\ddot{y}}]_i \tag{24}$$

Similarly, the acceleration at the end of the normal time increment, including the coupling forces, can be computed by modifying equation (15) as

$$\{\ddot{y}\}_{i+1} = [M]^{-1}(\{F\}_{i+1} - [C]\{\dot{y}\}_{i+1} - [C_c][\tilde{\dot{y}}]_{i+1} - [K]\{y\}_{i+1} - [K_c][\tilde{y}]_{i+1}) \tag{25}$$

With these modifications to the well-known formulae of the Wilson-θ method the total response of each foundation including through-the-soil coupling can be computed. Apparently, the modifications introduced by equations (24) and (25) do not alter the general flow of computations in the Wilson-θ method and, thus, existing computer codes require only minor changes.

As in any numerical method, the selection of the time step is critical to the accuracy of the solution of a step-by-step integration method. It is well established, see, for example, Karabalis and Beskos,[36] that using the Wilson-θ method accurate results can be obtained when the time interval is taken to be no longer than one-tenth of the natural period of the system being analysed. In addition, a sufficiently small time interval should be chosen such that the variation of the input load with time is properly represented. It was found in

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the course of this work that a time interval of one-twentieth of the lowest period of the coupled system was required to limit inaccuracies. Eigenvalue analyses using the Stodola method[33] were performed to determine the lowest period. The stiffness matrix necessary in these analyses was computed by a direct summation of the stiffness matrices of equations (16) and (18).

Computation of coupling coefficients

Having defined the time domain analysis procedure for incorporating through-the-soil coupling of foundations, the attention is now turned to the method used to determine the coupling functions for adjacent foundations. These functions depend on the distance between the foundations and the foundation size, and are necessary to compute the stiffness and damping coupling coefficients shown symbolically in equation (18).

The stiffness and damping coupling coefficients for each DOF are defined in their basic form in Table III. The reader will recognize that the equations in Table III are the same as those reproduced in Table I from Wolf[10] with the exception that the numerical constants are replaced by functions of the dimensionless distance ratio $d_{i,j}/a$, Γ for stiffness and Ψ for damping. These coupling functions were developed via an iterative process using the direct integration scheme described above for a two-foundation system in which one of the foundations was loaded and the responses of both foundations were computed. The off-diagonal stiffness and damping coupling coefficients were adjusted until the loaded and unloaded foundation responses fit existing time-domain solutions of the FSFI problem to within an average of 0·5 per cent error at each time step. In this particular work the calibration of the stiffness and damping functions Γ and Ψ was based on Huang's[25] time-domain BEM solutions for two identical, square foundations attached to an elastic half-space with Poisson ratio $v = \frac{1}{3}$. The Γ and Ψ functions produced by this iteration process correspond to dimensionless distance ratios of 0·25, 0·50, 1·0, 2·0 and 3·0 only and were verified for various foundation masses and soil material constants on the basis of the results provided by Huang.[25]

However, it would be useful, for apparent practical reasons, to develop continuous functions Γ and Ψ valid for any chosen distance ratio d/a. For this purpose standard regression analyses are used which, depending on the form of the functions Γ and Ψ, are based on logarithmic or exponential curve fitting formulae, of the form

$$\text{Logarithmic: } Y_i = \alpha * \mathrm{LOG}_{10}(X_i) \tag{26}$$

$$\text{Exponential: } Y_i = \alpha * 10^{(\varphi * X_i)} \tag{27}$$

where α and φ are constants and X_i represents the distance ratio d/a at which coupling constants have been computed from the above iterative process. These fitted curves allow the determination of coupling coefficients for any value of the distance ratio d/a, and also make possible the extrapolation of data beyond the distance ratios used in the development of the Γ and Ψ functions via the specific iteration process on the basis of the available numerical data.

The stiffness and damping coupling functions between any two foundations developed via the above described process are shown graphically in Figure 6 and given in an explicit form in Table IV. To apply these

Table III. Coupling coefficients for foundation–soil–foundation interaction

	Vertical motion	Horizontal motion	Rocking motion	Torsional motion
Stiffness	$\Gamma_3 \times \frac{Ga}{(1-v)}$	$\Gamma_1 \times \frac{Ga}{(2-v)}$	$\Gamma_{\phi 2} \times \frac{Ga^3}{(1-v)}$	$\Gamma_{\phi 3} \times Ga^3$
Damping	$\Psi_3 \times \frac{Ga^2}{V_S(1-v)}$	$\Psi_1 \times \frac{Ga^2}{V_S(2-v)}$	$\Psi_{\phi 2} \times \frac{Ga^4}{V_S(1-v)}$	$\Psi_{\phi 3} \times \frac{Ga^4}{V_S}$

Table IV. Explicit expressions of coupling functions for foundation–soil–foundation interaction

	Vertical motion	Horizontal motion	Rocking motion	Torsional motion
Stiffness, Γ	$1{\cdot}614 \times 10^{-0{\cdot}16257(d/a)}$	$3{\cdot}7561 \times 10^{-0{\cdot}18995(d/a)}$	$-(0{\cdot}04234 - 0{\cdot}2396 \times \mathrm{LOG}_{10}(d/a))$	$0{\cdot}05931$
Damping, Ψ	$8{\cdot}504$	$13{\cdot}2875$	$7{\cdot}3823 - 6{\cdot}775 \times \mathrm{LOG}_{10}(d/a)$	$4{\cdot}4429–2{\cdot}9125 \times \mathrm{LOG}_{10}(d/a)$

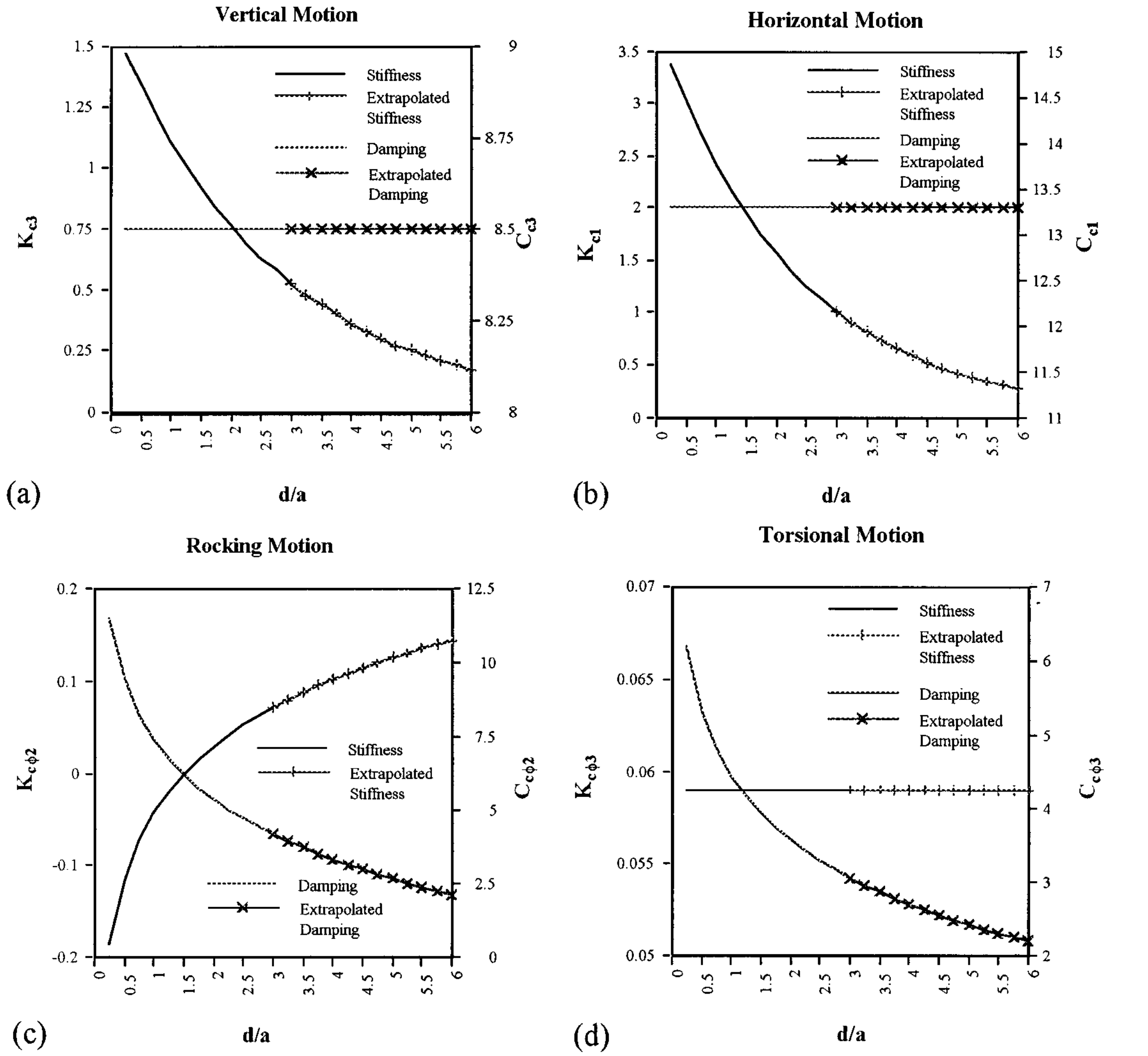

Figure 6. Coupling functions for: (a) vertical; (b) horizontal; (c) rocking and; (d) torsional motion versus distance ratio d/a

functions to multiple foundations, the stiffness and damping coupling coefficients between the first and second foundation, as shown in Figure 3, are computed by using the ratio $d_{1,2}/a$ in the coupling functions of Tables III and IV. Accordingly, $d_{2,3}/a$ is used for coupling between the second and third foundation, $d_{1,3}/a$ between the first and third foundation, etc. This procedure can be applied to any number of foundations in

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a row. It should be noted at this point, that the accuracy of the computed coupling functions between any two foundations is not, in general, affected appreciably by the existence of other foundations between them. This observation is supported by the detailed numerical results presented by Mohammadi and Karabalis[27,28] and is shown to hold true in the verification studies presented in the 'Numerical Results' section.

Near-field effects

During the development of the coupling functions proposed in this work, the influence of the near-field effect upon the accuracy of the results was also studied. It was determined, through extensive numerical verification studies,[34] that near-field effects are inconsequential for all but the rocking mode of vibration. Further analysis of this vibration mode indicated that the individual rocking stiffness and damping of each foundation was influenced by the close proximity of the adjacent foundations. In order to incorporate this effect for distance ratios $d/a \leqslant 0{\cdot}5$, stiffness and damping matrices in equation (16) are replaced by

$$[K] = \begin{bmatrix} K_1 + K_1^* & 0 & 0 & \dots \\ 0 & K_2 + K_2^* & 0 & \dots \\ 0 & 0 & K_3 + K_3^* & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$
$$[C] = \begin{bmatrix} C_1 + C_1^* & 0 & 0 & \dots \\ 0 & C_2 + C_2^* & 0 & \dots \\ 0 & 0 & C_3 + C_3^* & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{28}$$

where, upon computing the coupling coefficients, $K_{c(i,j)}$ and $C_{c(i,j)}$,

$$K_i^* = \sum_{j=1}^{\#\text{ of foundations}} K_{c(i,j)} \quad (i \neq j) \tag{29a}$$

$$C_i^* = \sum_{j=1}^{\#\text{ of foundations}} C_{c(i,j)} \quad (i \neq j) \tag{29b}$$

Equation (29) provides the means to incorporate on the independent stiffness and damping of each foundation the cumulative influence of all foundations adjacent to it.

It has also been determined that, in addition to the above modification to the stiffness and damping matrices, the stiffness and damping coupling functions for distance ratios $d/a \leqslant 0{\cdot}5$ are computed more accurately using the following expressions

$$\Gamma = -(0{\cdot}967 - 0{\cdot}880(d/a))$$
$$\Psi = 0.204 \tag{30}$$

However, it should be pointed out that the stiffness and damping functions for rocking vibrations shown in Table IV are still applicable for distance ratios $d/a > 0{\cdot}5$.

NUMERICAL RESULTS

This section provides numerical examples utilizing the methodology and coupling functions described in the previous sections of this work. First, results for a two-foundation system are presented for each DOF and

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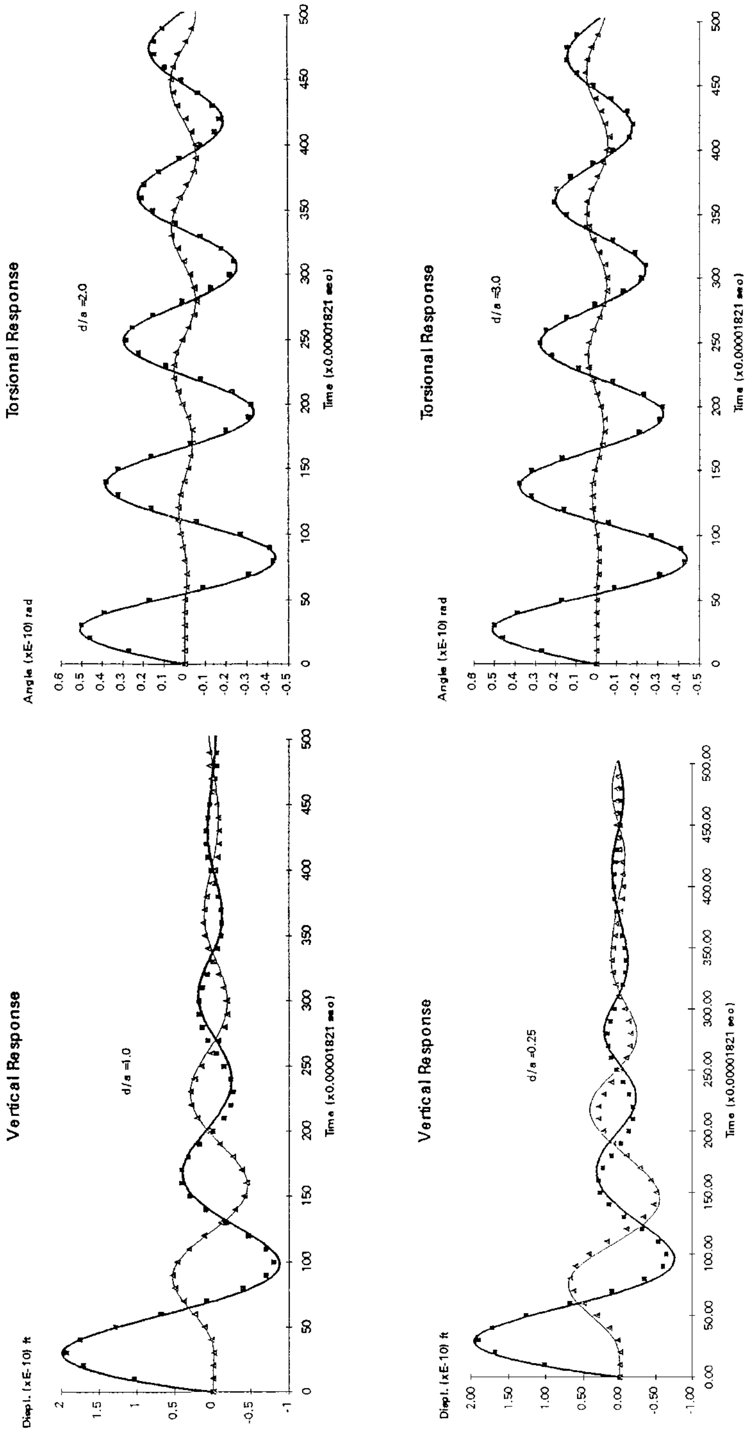
Vertical Response
Displ. (xE-10) ft
d/a =1.0
Time (x0.00001821 sec)
Torsional Response
Angle (xE-10) rad
d/a =2.0
Time (x0.00001821 sec)
Vertical Response
Displ. (xE-10) ft
d/a =0.25
Time (x0.00001821 sec)
Torsional Response
Angle (xE-10) rad
d/a =3.0
Time (x0.00001821 sec)

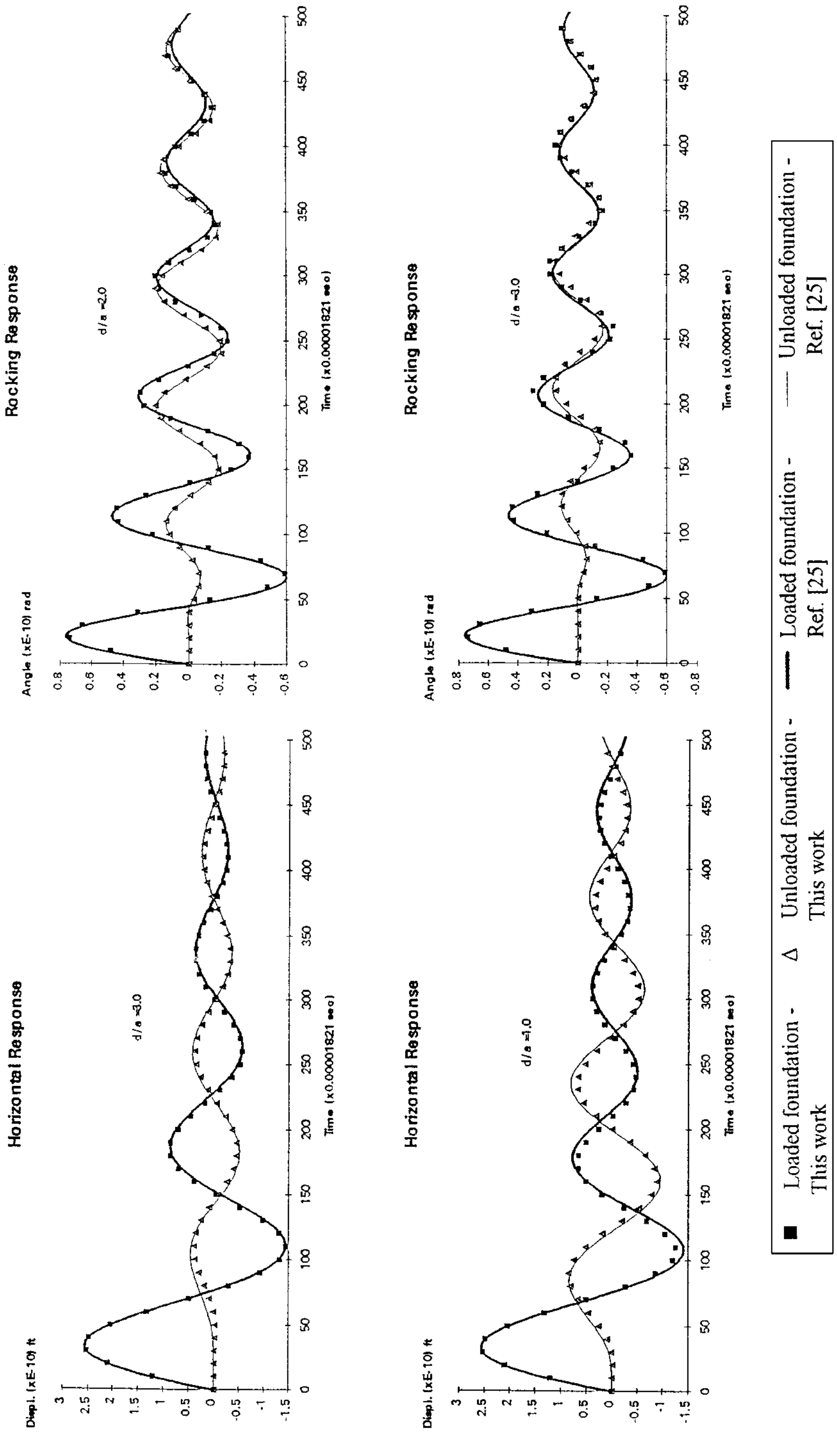

Figure 7. Loaded and unloaded foundation response versus time due to a rectangular impulse load

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various distance ratios, and comparisons are made against those BEM solutions used in the development of the coupling functions introduced in this work. Subsequently, in an attempt to establish the universal applicability of the proposed methodology, comparison studies with references to a Green's function solution of a two-foundation system and a finite element solution of a three-foundation system are also shown. Lastly, a study of the interaction of three adjacent structures undergoing earthquake excitation is presented by appropriately extending the methodology introduced in this work.

Two-foundation system

The dynamic response of two massive, rigid, square surface foundations on a homogeneous elastic half-space is studied first. The soil medium constants are: mass density, $\rho = 10{\cdot}368$ lb-sec^2/ft^4 (5343·5 kg/m^3), shear modulus, $G = 9{\cdot}71175 \times 10^5$ ksf (46·5 GPa), and Poisson's ratio, $\nu = \frac{1}{3}$. The foundations have a mass $M = 10 \times \rho r_0^3$ and a width $2a = 5$ ft (1·524 m). The external loads on one of the foundations are impulsive forces or moments, depending on the loaded DOF, in the form of a rectangular impulse with a magnitude of 100 lb (444·822 N) and a duration of 0·00001821 sec. Figure 7 provides a comparison of vertical, horizontal, torsional and rocking responses for various distance ratios of the results obtained using the proposed methodology and coupling functions and the BEM solutions of Huang.[25] The agreement between the two solutions is apparent. These comparison studies, although indicative of the accuracy of the proposed methodology, cannot be considered 'objective' since the results of Reference 25 have been used for the development of the coupling functions introduced in this work. Further similar studies of various distance and mass ratios using the proposed model can be found in Mulliken.[34]

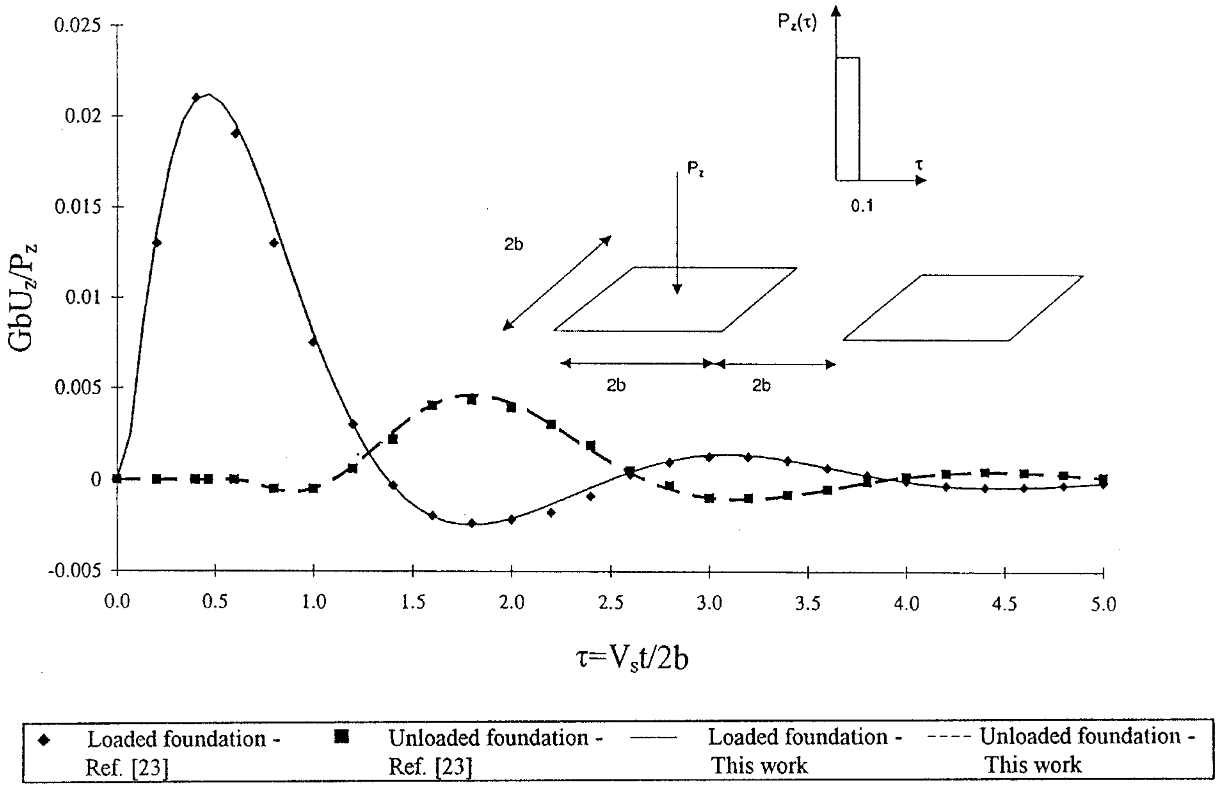

Figure 8. Non-dimensional loaded and unloaded foundation response versus time due to a rectangular impulse load

The results of the two-foundation system analysed by Guan and Novak[23] is used as an independent comparison study. In their study one foundation is loaded with a vertical force in the form of a rectangular impulse and the responses of both the loaded and unloaded foundations are computed in time domain on the basis of a Green's function formulation. A system of two rigid, square foundations each having mass $M = 1 \times \rho r_0^3$, and distance ratio $d/a = 2$ is used for the comparison study. Guan and Novak[23] provide their results in terms of dimensionless time $\tau = V_s t/2a$, and displacement $\delta = aGU_z/P_z$, where U_z is the foundation vertical displacement and P_z is the magnitude of the vertical impulse load as shown in Figure 8. The results from the proposed model and those from Guan and Novak[23] are shown in Figure 8 in dimensionless form. The close agreement of the results produced by the two methodologies should be noted.

Three-foundation system

To further demonstrate the accuracy of the proposed model, a finite element solution of a FSFI problem involving three foundations, geometrically arranged as shown is Figure 9, is performed with the computer program SASSI[37] and the results are compared to those obtained using the proposed model. The mechanical constants of the system are: mass of each foundation 1663·0124 lb-sec^2/ft (24269·84 kg), shear wave velocity of soil 600 ft/sec (182·88 m/sec), unit weight of soil 0·120 kips/ft^3 (18·85 kN/m^3), Poisson ratio of soil 1/3. A decaying sinusoidal forcing function is applied on foundation 1 only, as shown in Figure 9. Once again the close agreement of the results obtained by the two models is apparent.

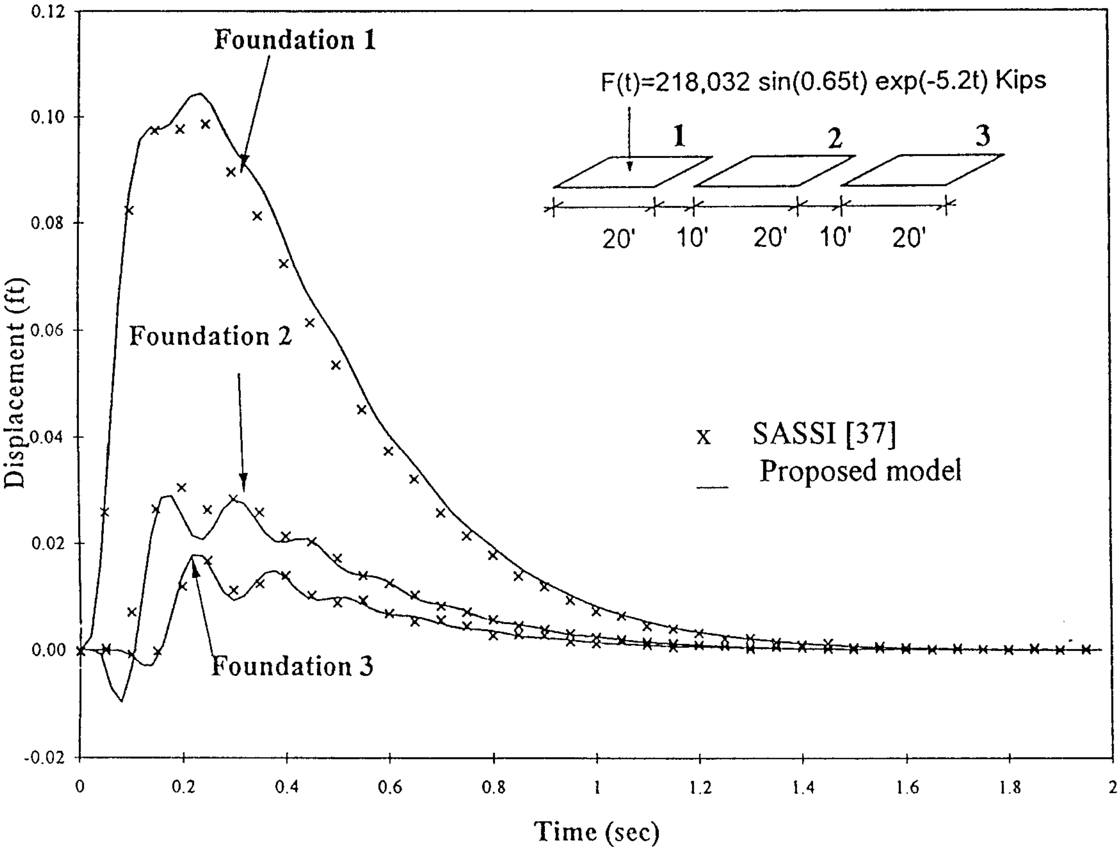

Figure 9. Dynamic response versus time of a set of three foundations due to an external impulsive load

Three-foundation–structure system

In this example the foundation coupling model developed in this work is extended to study the through-the-soil coupling of adjacent structures under earthquake induced excitation. The basic lumped parameter model of a single structure described by Wolf[10] is shown for completeness in Figure 10(a). This model considers the structure as a SDOF system that accounts for the fundamental vibration mode with the appropriate structural mass M_s, stiffness K_3 and damping C_3. Induced rocking motions are taken into account by supporting the structural model at a height h above the foundation by a rigid bar. The rigid bar is supported at the foundation level by translational and rocking springs and dashpots that are functions of the foundation size and the soil medium constants, as shown in Table I. By condensing the rotational DOF at the top of the rigid bar, a system of matrices for each structure is formulated as

$$[K^{**}]_i = \begin{bmatrix} K_1 + K_3 & hK_3 & -K_3 \\ hK_3 & K_2 + h^2K_3 & -hK_3 \\ -K_3 & -hK_3 & K_3 \end{bmatrix}$$

$$[C^{**}]_i = \begin{bmatrix} C_1 + C_3 & hC_3 & -C_3 \\ hC_3 & C_2 + h^2C_3 & -hC_3 \\ -C_3 & -hC_3 & C_3 \end{bmatrix} \tag{31}$$

$$[M^{**}]_i = \begin{bmatrix} M_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & M_s \end{bmatrix}$$

where the subscript 'i' indicates the foundation number, and $[K^{**}]_i$, $[C^{**}]_i$ and $[M^{**}]_i$ operate on the displacement, velocity and acceleration vectors, respectively,

$$\{Y^{**}\}_i = \begin{Bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{Bmatrix}, \qquad \{\dot{Y}^{**}\}_i = \begin{Bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \end{Bmatrix} \quad \text{and} \quad \{\ddot{Y}^{**}\}_i = \begin{Bmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \\ \ddot{y}_3(t) \end{Bmatrix} \tag{32}$$

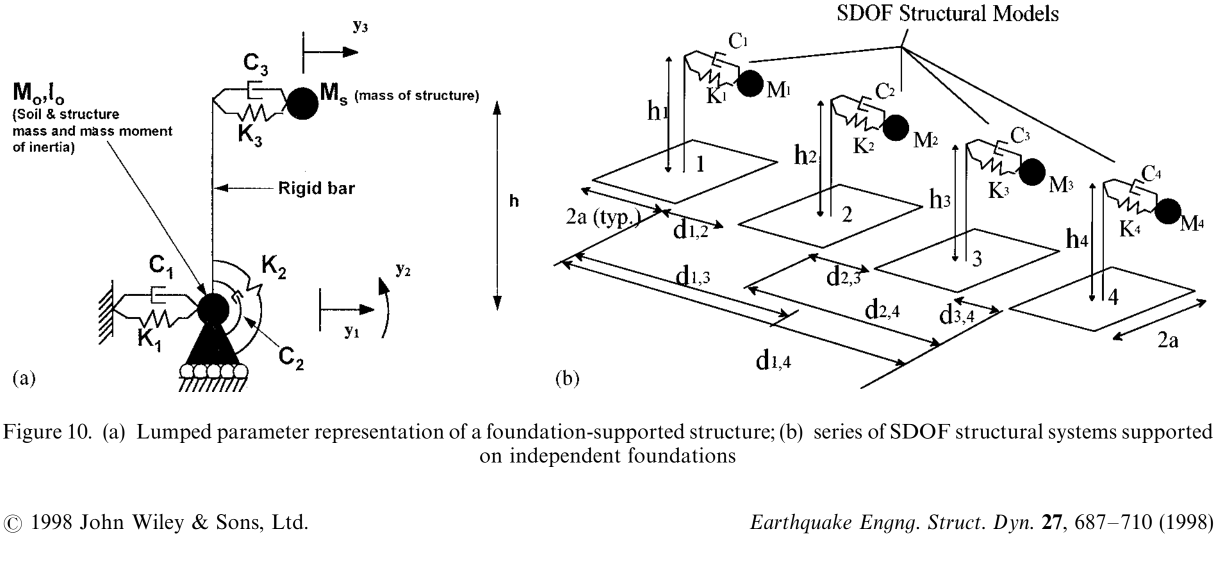

Figure 10. (a) Lumped parameter representation of a foundation-supported structure; (b) series of SDOF structural systems supported on independent foundations

where the subscripts 1, 2 and 3 indicate the three DOFs at each foundation supported structure as shown in Figure 10(a). The earthquake forces are computed by multiplying the structural and soil masses by the earthquake acceleration time history, $\ddot{a}_g(t)$

$$\{F^{**}\}_i = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} = \begin{Bmatrix} -M_0\ddot{a}_g(t) \\ 0 \\ -M_s\ddot{a}_g(t) \end{Bmatrix} \tag{33}$$

The consideration of multiple structures, as shown in Figure 10(b), is achieved by coupling the foundations' translational and rocking DOFs using the coupling functions and the direct integration procedure proposed in this work. The procedure presented for multiple foundations is also utilized here with the following changes to the stiffness, damping and mass matrices of equation (16) in order to incorporate the building properties

$$K = \begin{bmatrix} [K^{**}]_1 & 0 & 0 & \cdots \\ 0 & [K^{**}]_2 & 0 & \cdots \\ 0 & 0 & [K^{**}]_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$C = \begin{bmatrix} [C^{**}]_1 & 0 & 0 & \cdots \\ 0 & [C^{**}]_2 & 0 & \cdots \\ 0 & 0 & [C^{**}]_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{34}$$

$$M = \begin{bmatrix} [M^{**}]_1 & 0 & 0 & \cdots \\ 0 & [M^{**}]_2 & 0 & \cdots \\ 0 & 0 & [M^{**}]_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where the subscripts indicate the foundation number. The displacement, velocity and acceleration vectors of equation (17) are also re-written to include the structural behaviour

$$y = \begin{Bmatrix} \{Y^{**}\}_1 \\ \{Y^{**}\}_2 \\ \{Y^{**}\}_3 \\ \vdots \end{Bmatrix}, \quad \dot{y} = \begin{Bmatrix} \{\dot{Y}^{**}\}_1 \\ \{\dot{Y}^{**}\}_2 \\ \{\dot{Y}^{**}\}_3 \\ \vdots \end{Bmatrix}, \quad \ddot{y} = \begin{Bmatrix} \{\ddot{Y}^{**}\}_1 \\ \{\ddot{Y}^{**}\}_2 \\ \{\ddot{Y}^{**}\}_3 \\ \vdots \end{Bmatrix}, \quad F = \begin{Bmatrix} \{F^{**}\}_1 \\ \{F^{**}\}_2 \\ \{F^{**}\}_3 \\ \vdots \end{Bmatrix} \tag{35}$$

The coupling matrices of equation (18) are also expanded to handle both translational and rocking vibrations at each foundation

$$K_c = \begin{bmatrix} 0 & [K_c^{**}]_{1,2} & [K_c^{**}]_{1,3} & \cdots \\ [K_c^{**}]_{2,1} & 0 & [K_c^{**}]_{2,3} & \cdots \\ [K_c^{**}]_{3,1} & [K_c^{**}]_{3,2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad C_c = \begin{bmatrix} 0 & [C_c^{**}]_{1,2} & [C_c^{**}]_{1,3} & \cdots \\ [C_c^{**}]_{2,1} & 0 & [C_c^{**}]_{2,3} & \cdots \\ [C_c^{**}]_{3,1} & [C_c^{**}]_{3,2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{36}$$

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where

$$[K_c^{**}]_{i,j} = [K_c^{**}]_{j,i} = \begin{bmatrix} -Kh_{c(i,j)} & 0 & 0 \\ 0 & -Kr_{c(i,j)} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[C_c^{**}]_{i,j} = [C_c^{**}]_{j,i} = \begin{bmatrix} -Ch_{c(i,j)} & 0 & 0 \\ 0 & -Cr_{c(i,j)} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{37}$$

with $Kh_{c(i,j)}$ and $Ch_{c(i,j)}$ being the horizontal coupling coefficients and $Kr_{c(i,j)}$ and $Cr_{c(i,j)}$ the rocking coupling coefficients.

Finally, the time lagging of coupled foundation input is incorporated by the modification of the matrices in equations (20)–(22). As previously discussed, the first vector in each matrix contains the displacements at all foundations adjacent to foundation 1 that are lagged by the distances between these foundations and foundation 1, the second vector provides these values in reference to foundation 2, etc. Similar, considerations should be made for the multiple structure system in order to appropriately include through-the-soil coupling. Since there are now three DOFs at each foundation supported structure, as identified in Figure 10(a), the following substitutions are made in equations (20)–(22):

$$y_j(t_k - \Delta T_{i,j}) = \begin{bmatrix} y_{1,j}(t_k - \Delta T_{i,j}) & 0 & 0 \\ 0 & y_{2,j}(t_k - \Delta T_{i,j}) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{y}_j(t_k - \Delta T_{i,j}) = \begin{bmatrix} \dot{y}_{1,j}(t_k - \Delta T_{i,j}) & 0 & 0 \\ 0 & \dot{y}_{2,j}(t_k - \Delta T_{i,j}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{38}$$

$$\ddot{y}_j(t_k - \Delta T_{i,j}) = \begin{bmatrix} \ddot{y}_{1,j}(t_k - \Delta T_{i,j}) & 0 & 0 \\ 0 & \ddot{y}_{2,j}(t_k - \Delta T_{i,j}) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where the subscript 'k' indicates the current time increment, 'i' is the foundation at which we are computing coupling forces and 'j' represents the foundations adjacent to foundation 'i'. The subscripts '1,j' and '2,j' indicate the horizontal and rocking DOFs of foundation 'j'.

Next, a numerical example utilizing the above procedure is presented. Three structures, each having the same stiffness of 66 000 kip/ft (963·2 MN/m), mass of 26 kip-sec^2/ft (379441.7 kg), 7 per cent of critical damping and height above the foundation of 35 ft (10·668 m), are supported on independent rigid surface foundations with half-widths $a = 20$ ft (6·096 m). The separation between foundations is 10 ft (3·048 m). The soil medium constants are: shear modulus = 360 ksf (17·237 MPa), mass density = 3·996 lb-sec^2/ft^4 (2059·45 kg/m^3) and Poisson ratio $= \frac{1}{3}$. To begin the study, sinusoidal acceleration time histories of unit magnitude and frequency range 0–16 Hz, are input to a single foundation–structure system and a three foundation–structure system, as described above. The peak response at each frequency for both systems is shown in Figure 11. It is apparent that each of the structures in the three foundation system experiences

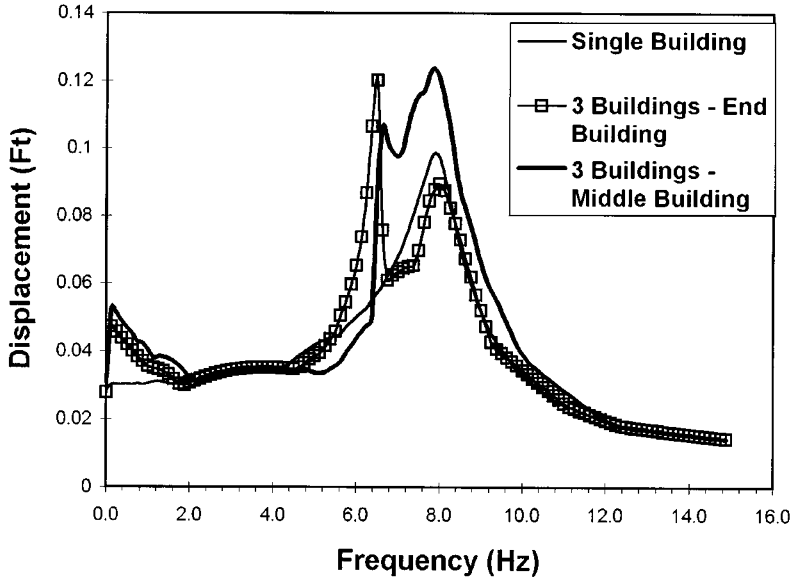

Figure 11. Structural response versus frequency for single- and multiple-building systems

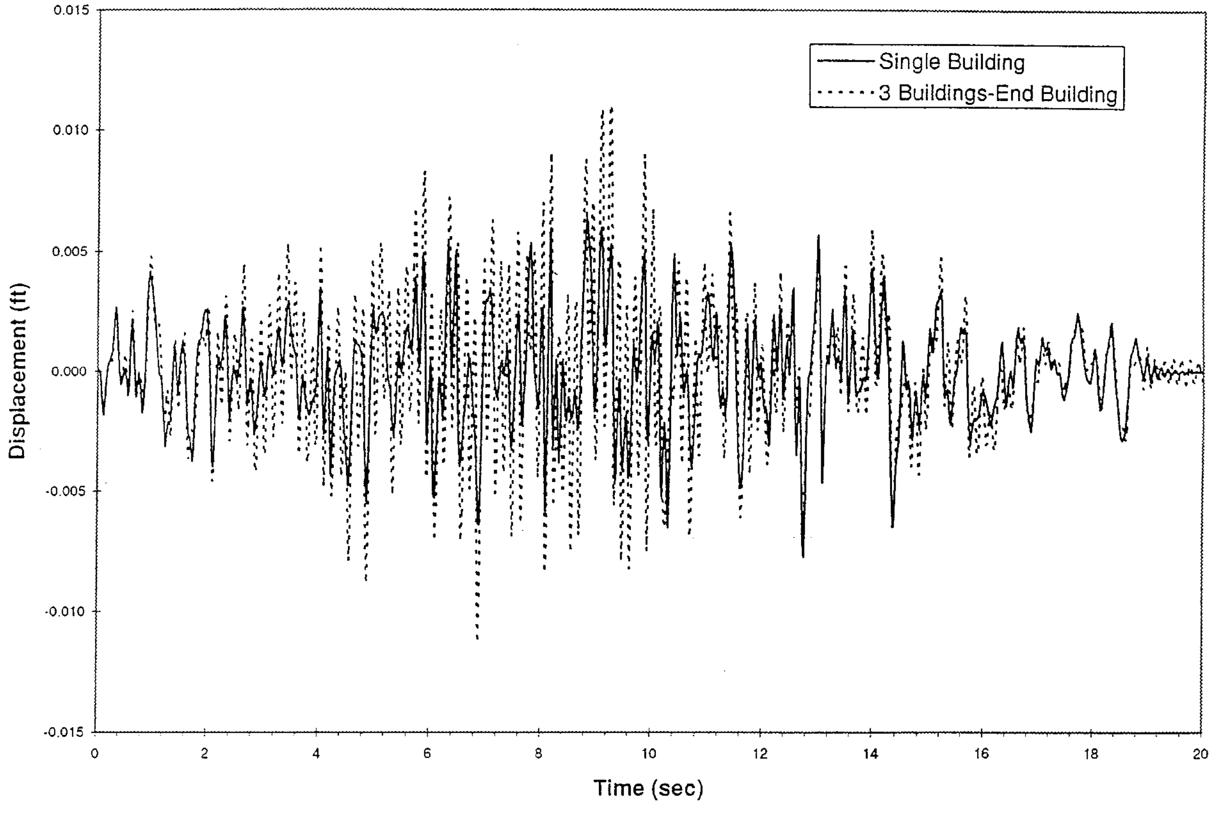

Figure 12(a). Seismic response versus time for a single-building and the end building of a three-building system

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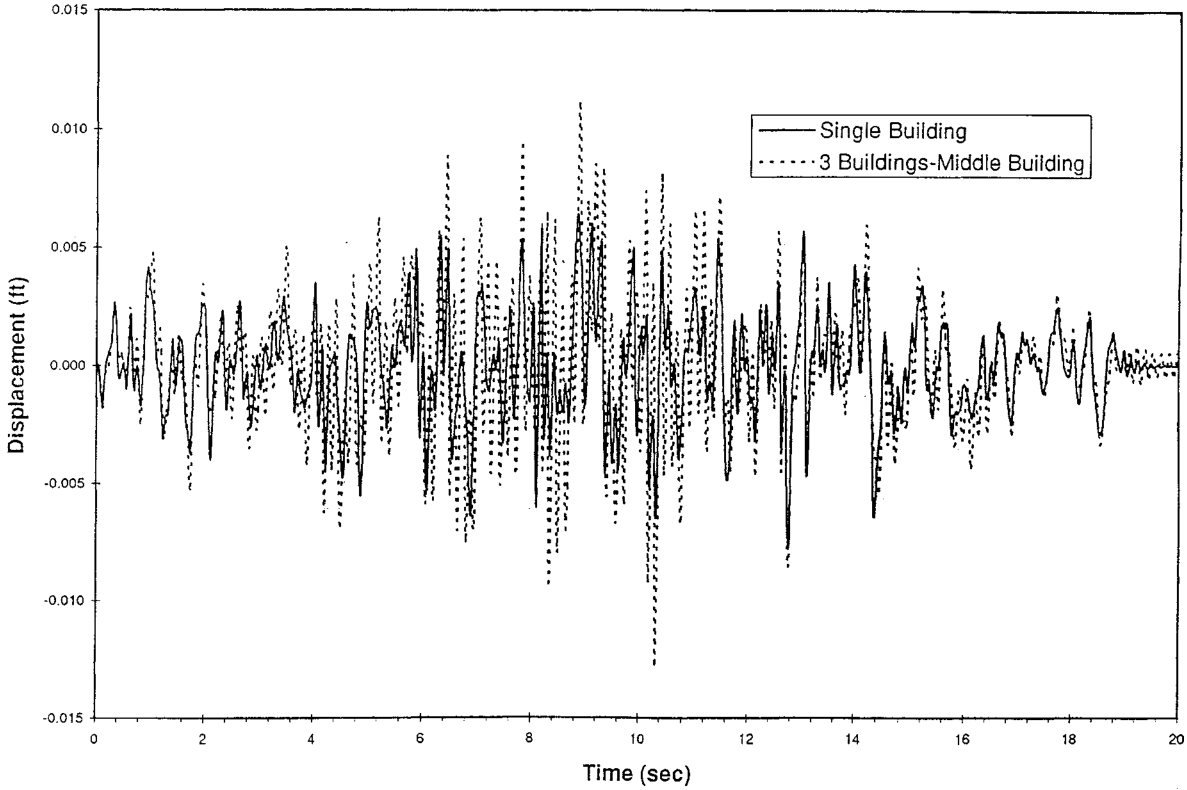

Figure 12(b). Seismic response versus time for a single building and the middle building of a three-building system

motions and 'resonance' peaks that are significantly altered by the presence of the adjacent structures. Although for this particular example this observation appears to be partially explained by the fact that the coupled dynamic system frequency is near the fixed base natural frequency of the single structure, it also demonstrates the importance of the FSFI phenomenon.

The time-domain responses of the single- and three-building systems are also computed given an artificial earthquake input motion with a peak ground acceleration of $0{\cdot}2g$. The response of the single structure is compared to the end structure of the three-building system in Figure 12(a) and the middle structure in Figure 12(b). It is evident again that large differences in response can be produced due to the presence of adjacent structures.

CONCLUSIONS

Combining classical foundation–soil interaction models and the coupling models developed in this work, the foundation–soil–foundation interaction of a number of adjacent rigid surface foundations can be computed. This approach takes advantage of available data for foundation–soil–foundation dynamic interaction and includes familiar terminology and methodologies of dynamic lumped parameter systems for incorporating the dynamic through-the-soil coupling of adjacent foundations and structures. The numerical integration technique utilized can also be modified to include the effects of non-linear soil and structures. Through a variety of comparison studies performed in this work the proposed methodology for dynamic foundation–soil–foundation interaction is proven accurate, efficient and applicable to the evaluation of linear and non-linear systems. A modification of the above basic foundation–soil–foundation model is also used to

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study dynamic structure-soil–structure interaction phenomena. These studies have demonstrated the direct applicability of the basic model to more complicated soil-structure systems and the possible importance of the dynamic through-the-soil interaction phenomenon in structural design.

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